Direct Numerical Simulation of Driven Cavity Flows

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Direct Numerical Simulation of Driven Cavity Flows

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Abstract. Direct numerical simulations of 2D driven cavity flows have been performed. The simulations exhibit that the flow converges to a periodically oscillating state at Re=11,000, and reveal that the dynamics is chaotic at Re=22,000. The dimension of the attractor and the Kolmogorov entropy have been computed. Explicit time-integration techniques are discussed.

Key words: DNS – periodic and chaotic driven cavity flows – explicit time-integration

1. Introduction

Direct numerical simulations (DNS) provide data for turbulence reseach. Data that describes the evolution of all scales of motion. Such data can only be computed with high resolution and fully reliable, accurate numerical methods. The resolution requirements limit DNS to low Reynolds numbers: the computing resources are simply insufficient for high *Re*. The teraflop challenge holds the promise of simulating turbulence at higher Reynolds numbers. The numerical challenge is to develop numerical simulation methods that are more reliable, accurate, efficient and geometrically flexible than today's methods and thus enlarge the number of flows emenable to DNS. Hereto we persue a step-by-step evaluation of numerical discretisations and time-integration techniques. Starting with simple 1D applications and finally resulting in 3D configurations the steps act as selection filter for numerical methods.

Here, we focus on the 2D square, lid-driven cavity. The driven cavity is a well-known benchmark problem for numerical methods for laminar flows. For example, Shyy *et al.* [1] used laminar driven cavity flows to evaluate difference schemes for the convective terms in the Navier-Stokes equations. Steady state driven cavity flows have been reported for Reynolds' numbers up to 10,000, e.g. in [2]. Unsteady, oscillating, cavity flows can be found in the references [3], [4] and [5]. Gustafson and Halasi [3] presented a numerical study of the first Hopf bifurcation (at *Re*=10,000) of the flow in a rectangular driven cavity with aspect ratio 2. Shen [4] computed periodic and two-periodic (*Re*=15,500) flows in a so-called regularized, square driven cavity. Recently, Huser and Biringen [5] have computed unsteady oscillating flows with one fundamental frequency in a square cavity, where the flow is driven by a constant surface velocity gradient.

2. Computational procedure

The incompressible Navier-Stokes equations are solved on a regular staggered 333*333 grid using the Marker-and-Cell method [6]. The pressure gradient and the incompressiblility constraint are treated implicitly; the convective and viscous terms are treated explicitly. The spatial discretisations are second-order accurate. The discrete Poisson equation for the pressure is solved using the conjugate gradient method with modified incomplete Choleski preconditioning. The ICCG code is fully vectorized by explicitly reordening the unknowns along diagonals of the grid, and is optimized using Eisenstat's implementation (cf. [7] and references therein). The convergence criterion is such that the resulting divergence of the velocity field is two orders of magnitude smaller than the discretisation error. The CPU-time for the numerical solution of the Navier-Stokes equations on a Cray Y-MP (one processor) is 2.4 μs per gridpoint and time-step.

3. Explicit time-integration techniques

3.1. Stability versus accuracy

Both accuracy and stability limit the time-step of explicit time-integration methods for DNS. Resolving the dynamics at the smallest time scale τ and the smallest length scale l requires, say, n_t time-steps Δt per τ and n_x steps of size Δx per l, where n_t and n_x depend on the accuracy of the applied temporal and spatial numerical discretisation techniques (see e.g. [8] for resolution requirements). Explicit time-integration of the Navier-Stokes equations is stable if the time-step satisfies both $2\nu\Delta t < \Delta x^2$ and the CFL-condition $U\Delta t < \Delta x$. Here, ν denotes the viscosity and U is the maximum velocity. To compare these two time-step limits we take here, as in 3D homogeneous, isotropic turbulence, [1] the ratio of the time scale of the largest eddies to τ to be $Re^{0.5}$ and the ratio of the length of the largest eddies to l to be $Re^{0.75}$, where the Reynolds number Re is based on the velocity (u) and length scales of the large eddies. The stability restrictions can then be expressed in terms of n_t and n_x:

$$n_t > 2n_x^2 \quad \text{and} \quad n_t > \frac{U}{u} Re^{\frac{1}{4}} n_x.$$

The latter constraint, the CFL-condition, is active if the eddies deform at a rate significantly smaller than the maximum velocity U. This is the case for channel flows, in a boundary layer over a flate plate, etc. Then, the treatment of the convective terms in the Navier-Stokes equations determines the stability of the numerical approach. Furthermore, the number n_t of time-steps per smallest time scale τ is likely to be determined by the CFL-condition, and not by accuracy; e.g. for Re=500, U/u=10 and n_x=5 the CFL-condition states that $n_t > 236$.

[1] Other cases, with different ratios between the largest and smallest scales, can be considered analogously

3.2. ADAMS-BASHFORTH AND RUNGE-KUTTA METHODS

Starting from a statistical equilibrium, the Navier-Stokes equations for 2D incompressible flow (*Re*=22,000) in a driven cavity were integrated over 20 large-eddy turn-around-times using four different explicit methods: (a) 2nd order accurate Adams-Bashforth, (b) modified Euler, (c) a 3rd order, 3-stage Runge-Kutta method with zero entries in the lower triangle of the Butcher array, and (d) the 2nd order, 4-stage Runge-Kutta method with 6th order phase error of Van der Houwen and Sommeijer [9]. The resulting relative differences in the energy, dissipation, enstrophy, palenstrophy, wall friction, Reynolds stresses and fluctuating velocities were smaller than 0.01%! Furthermore, the computations manifested that the maximum allowed time-step is closely related to the length of the imaginary stability interval of the time-integration method. The computation times of the four methods did not differ significantly when the maximum allowed time- steps are used. In conclusion: the computations with the four different explicit time-integration methods confirm that (*i*) convection determines the stability and (*ii*) the time-step is determined by stability and not by accuracy.

4. Results and discussion

4.1. THE FIRST BIFURCATION

The square, driven cavity is a point of reference in evaluating numerical methods for the steady Navier-Stokes equations. Converged numerical solutions have been reported for $Re \leq 10,000$. Our numerical integration of the time-dependent Navier-Stokes equations, taking a previously computed steady state (at *Re*=5,000) as initial condition, showed that the flow converges to a periodic state at *Re*=11,000. Without triggering, it took approximately 2000 seconds (10^6 time-steps) to approach a statistically steady balance between the dissipation and production of energy. The amplitude of the periodic motion is small: in terms of the total kinetic energy 0.2% of the mean value. The period is 1.8 seconds. Figure 1 shows the origination, motion and merging of eddies near three corners of the cavity during one full period.

4.2. CHAOS AT RE=22,000

The driven cavity flow at *Re*=11,000 is probably not suited for evaluation of numerical methods for DNS. For that, the flow is too regular. Therefore, we have computed unsteady driven cavity flows at higher Reynolds numbers. The figures 2 and 3 display results for *Re*=22,000. As expected, the number of vortices has somewhat increased compared to the simulation at *Re*=11,000. The amplitude of the oscillation has increased too: the kinetic energy fluctuates between 0.032 and 0.035. Figure 3 reveals the seemingly chaotic motion of vortical structures close to the bottom wall. Also, the time-series obtained from the simulation at

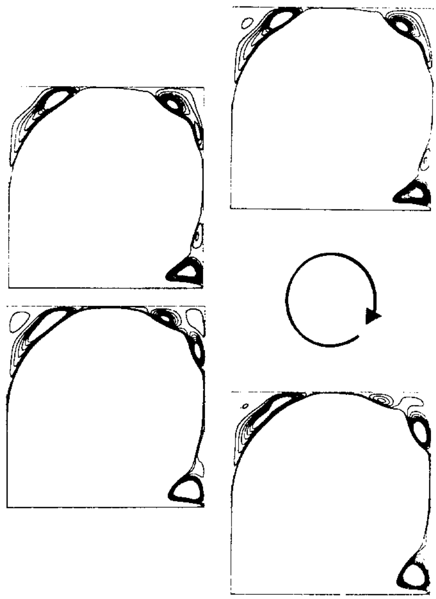

Fig. 1. Streaklines at Re=11,000 showing the dynamics of small eddies during one period.

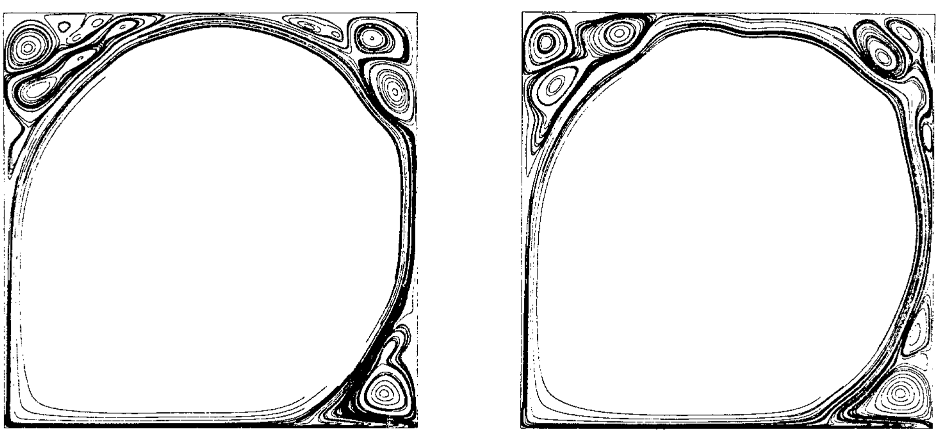

Fig. 2. Two snapshots of streaklines at Re=22,000.

Re=22,000 exhibits chaotic behaviour. We employed the method proposed in [10] to analyse the time-series of the energy dissipation. The analysed part of the time-series consisted of 24,000 terms. The (correlation) dimension of the attractor is computed from the correlation integral: the dimension is approximately 2.8. The Kolmogorov entropy K is also estimated directly from the correlation integral. A

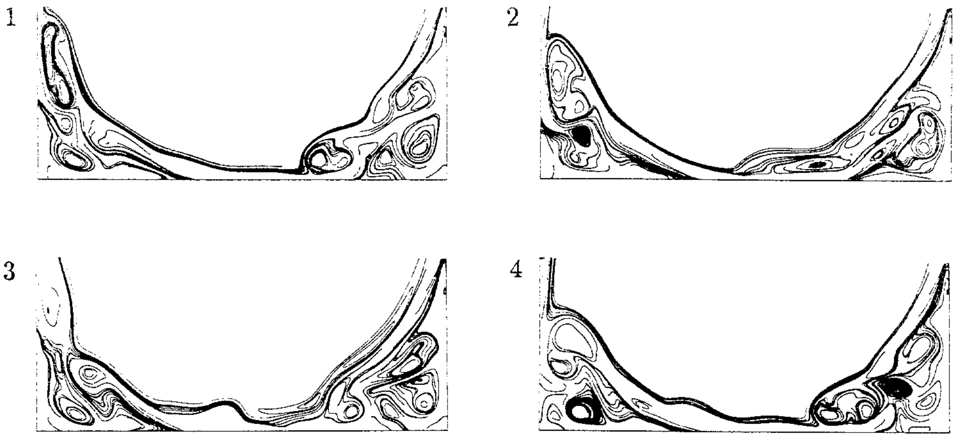

Fig. 3. Motion of vortical structures at the bottom of the cavity for Re=22,000.

chaotic (deterministic) system is characterized by $0 < K < \infty$, an ordered system by $K = 0$, and a random system by $K = \infty$. Here, we found $K \approx 3$. Thus, the dynamical behaviour is, indeed, chaotic.

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